AN EXPERIMENTAL INVESTIGATION OF PULSATING SUBSONIC FLOW IN A CONICAL NOZZLE.

Edward James Carlson

United States
Naval Postgraduate School

THESIS

AN EXPERIMENTAL INVESTIGATION OF
PULSATING SUBSONIC FLOW IN
A CONICAL NOZZLE

by

Edward James Carlson

Thesis Advisor: Thomas M. Houlihan

September 1971

An Experimental Investigation of Pulsating Subsonic Flow in a Conical Nozzle

by

Edward James Carlson
Lieutenant, United States Navy
B.S., United States Naval Academy, 1966

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
September 1971

ABSTRACT

Amplitude and phase relationships between sinusoidal pressure disturbances in the exit plane of a conical nozzle and mass flux response were experimentally determined for the subsonic flow of air in the nozzle. The experimental results were compared with values predicted by using the modified asymptotic WKB method.

Experimental measurements were made with a constant temperature hot wire anemometer and a condenser microphone. Disturbance frequencies from 20 to 250 Hz were investigated for Mach numbers up to .34.

The experimental values of amplitude factor agree quite well with analytical predictions for all Mach numbers tested. Experimental values for phase angle differed from those predicted but exhibited a similar trend.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

NOMENCLATURE

a	speed of sound
a_o	speed of sound at stagnation conditions
A	flow area
A^*	flow area where $M = 1$
C_1, C_2	arbitrary constants
$\frac{D}{Dt}$	substantial derivative
F	defined in Eq. (24)
g	defined in Eq. (29)
h	enthalpy
H	stagnation enthalpy
i	$\sqrt{-1}$
M	Mach number
N_1, N_2, N_3, N_4	defined in Eq. (62)
$\overline{N}$	amplitude factor
p	Pressure
q	velocity (magnitude)
$\overline{q}$	velocity vector
r	radius measured from apex of cone
R	radius where $M = 1$
s	entropy
S^*	modified Strouhal number $\omega r/q$
T	temperature
T_o	stagnation temperature
t	time
u_1, v_1	real and imaginary components of Z_1

u_2, v_2	real and imaginary components of Z_2
V	hot wire voltage
V_o	hot wire zero voltage
w	frequency number $\omega R/a_o$
x	$= M^2$
y	defined in Eq. (65)
Z_1, Z_2	defined in Eq. (34)
α	q/q_o
β	a/a_o
γ	ratio of specific heats (= 1.4 for air)
ζ	r/R
η	defined in Eq. (25)
μ	$(\rho q)'/\rho q$
ξ	transformed ζ
π	p'/p
ρ	density
τ	dimensionless time $a_o t/R$
ϕ	dimensionless enthalpy perturbation H'/a_o^2
Φ	$\frac{dx}{d\xi}$
$\overline{\Psi}$	phase angle
ω	angular frequency

ACKNOWLEDGEMENT

The author wishes to express his sincere appreciation to Professor T. Houlihan for his guidance and advice during the course of this investigation. Appreciation is also expressed to Messrs. K. Mothersell, J. McKay, and J. Beck for their work in the construction of the test apparatus and to Mr. T. Christian for his able assistance with the electronic instrumentation.

I. INTRODUCTION

The engineering application of nozzles in the control of fluid flow is extremely widespread. It ranges in scope from small fluidic devices to large rocket thrusters. In the steady state there exists, for a given absolute pressure ratio across the nozzle, a corresponding mass flow rate. The introduction of pressure fluctuations at the nozzle boundaries affects the flow and certain dynamic characteristics are exhibited for a given set of conditions, i.e., nozzle geometry, type of fluid medium, and the nature of the pressure fluctuations. Because such fluctuations are quite common, especially in systems possessing cyclic operating components, a knowledge of the dynamic characteristics becomes important. In the past, a quasi-steady relationship between mass flow rate and pressure fluctuations was assumed. Thus, an unsteady situation was reduced to a sequence of instantaneous steady state correspondences. The validity of this assumption is questionable.

Oppenheim and Chilton [3] conducted a literature survey on pulsating flow measurement. Most of the investigations noted had to do with the effect of pressure fluctuations on the time average of flow. Elrod [1], working in connection with a stability analysis of externally pressurized thrust bearings, performed a frequency response analysis to predict the amplitude and phase relations between the instantaneous flow rate and a sinusoidally fluctuating exit pressure in an infinite conical nozzle. Although the chosen geometry was extremely simple, Elrod [1] stated "It is believed that the essential characteristics of a compressible fluid expanding and accelerating from a stagnation reservoir are well enough realized for the results to have semi-quantitative practical significance." He further assumed that the flow was adiabatic,

frictionless, and irrotational. The analysis using the WKB method is asymptotically valid for high frequency number $W=\omega R/a$ and low Mach number. Unfortunately, the numerical results were wrong because of a wrong choice of sign for the divergence term of the developing equations.

Chiang, et al., [2], realizing the generality of Elrod's analysis (it can be applied to either converging or diverging flow), corrected the choice of sign and extended the region of validity to lower frequency and higher Mach number by modifying the WKB method. They also investigated the problem of pulsating flow in a finite conical nozzle.

The objective of this thesis was to experimentally investigate the mass flux - pressure relationship for pulsating subsonic flow in a conical nozzle and to compare experimental results with values predicted using the modified WKB solution.

II. THEORETICAL ANALYSIS

Elrod [1], in his original analysis, assumed an adiabatic, friction free (i.e., isentropic) flow originating in a stagnation reservoir. Under these conditions, the flow remains irrotational in accordance with Kelvin's theorem and the following governing equations pertain:

Momentum

$$\frac{D\bar{q}}{Dt} + \frac{1}{\rho}\nabla P = 0 \tag{1}$$

Continuity

$$\frac{D\rho}{Dt} + \rho\nabla\cdot\bar{q} = 0 \tag{2}$$

which can also be written as

$$\frac{D \ln \rho}{Dt} + \nabla \cdot \bar{q} = 0 \tag{3}$$

Gibb's Equation for a simple compressible substance

$$T\,ds = dh - \frac{1}{\rho}\,dp = 0 \qquad \text{(isentropic)} \tag{4}$$

For irrotational flow:

$$(\bar{q} \cdot \nabla)\bar{q} = \frac{1}{2}\nabla(\bar{q} \cdot \bar{q}) = \nabla \frac{q^2}{2} \tag{5}$$

(see Robertson [4], p. 46.)

Using Eq. (5), Eq. (1) can be rewritten as

$$\frac{\partial \bar{q}}{\partial t} = -\frac{1}{\rho}\nabla p - \nabla \frac{q^2}{2} \tag{6}$$

Stagnation enthalpy is defined as

$$H = h + \frac{q^2}{2} \tag{7}$$

Equations (4), (6), and (7) combine to yield

$$\frac{\partial \bar{q}}{\partial t} = -\nabla H \tag{8}$$

The scalar product of $\bar{q}$ with Eq. (1), together with Eq. (4), gives

$$\frac{D\frac{q^2}{2}}{Dt} = -\frac{1}{\rho}\,\bar{q} \cdot \nabla p = -\bar{q} \cdot \nabla h = -\frac{Dh}{Dt} + \frac{\partial h}{\partial t} \tag{9}$$

which can also be written as

$$\frac{DH}{Dt} = \frac{1}{\rho}\frac{\partial p}{\partial t} \tag{10}$$

by considering a time derivative form of Eq. (4).

The acoustic velocity is defined by

$$a = \left[\left(\frac{\partial p}{\partial \rho}\right)_s\right]^{\frac{1}{2}} \tag{11}$$

Therefore, by the chain rule, Eq. (10) becomes

$$\frac{1}{a^2}\frac{DH}{Dt} = \frac{1}{\rho}\frac{\partial \rho}{\partial t} = \frac{\partial \ln \rho}{\partial t} \tag{12}$$

Taking the total derivative of Eq. (12) gives

$$\frac{D}{Dt}\left[\frac{1}{a^2}\frac{DH}{Dt}\right] = \frac{D}{Dt}\,\frac{\partial \ln \rho}{\partial t} \tag{13}$$

but

$$\begin{aligned}\frac{D}{Dt}\frac{\partial \ln \rho}{\partial t} &= \frac{\partial}{\partial t}\frac{\partial \ln \rho}{\partial t} + \frac{\partial}{\partial t}\left(\bar{q}\cdot\nabla \ln \rho\right) - \frac{\partial \bar{q}}{\partial t}\cdot\nabla \ln \rho \\ &= \frac{\partial}{\partial t}\left(\frac{D \ln \rho}{Dt}\right) - \frac{\partial \bar{q}}{\partial t}\cdot\nabla \ln \rho\end{aligned} \tag{14}$$

Substituting Eqs. (3), (8), and (14) into Eq. (13) above produces Elrod's [1] "propagation equation" for stagnation enthalpy:

$$\frac{D}{Dt}\left[\frac{1}{a^2}\frac{DH}{Dt}\right] = \nabla^2 H + \nabla H\cdot\nabla \ln \rho \tag{15}$$

For spherically symmetric flow in a conical passage with slight flow deviations which are also spherically symmetric, the flow is described by only one spatial coordinate - namely the radius r originating

at the apex of the cone.

Chiang, et al., [2] used linear perturbation theory to express a quantity as the sum of a steady-state value and a perturbation.

Stagnation enthalpy was written as

$$H = \bar{H} + H' \tag{16}$$

where the prime denotes a perturbation term.

Since the stagnation enthalpy is uniform throughout the mean flow, it is apparent that

$$\nabla \bar{H} = 0 \tag{17}$$

and

$$\frac{D\bar{H}}{Dt} = 0 \tag{18}$$

Hence, Eq. (15) may be written, to the first order in small quantities, as

$$\frac{D}{Dt}\left[\frac{1}{a^2}\frac{DH'}{Dt}\right] = \nabla^2 H' + \nabla H' \cdot \nabla \ln \rho \tag{19}$$

As mentioned previously, spherically symmetrical flow is one-dimensional. Chiang, et al., [2] pointed out that for this case

$$\nabla = \frac{\partial}{\partial r}$$

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right) \tag{20}$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} \pm q \frac{\partial}{\partial r}$$

The upper sign pertains to diverging flow and the lower sign is related to converging flow. This choice of signs is based upon the fact that for diverging flow the velocity and increasing radius are

identical in direction. For converging flow, the directions are opposed. Therefore, the lower (negative) sign applies to nozzle flows.

For convenience, Elrod [1] introduced the following dimensionless variables:

$$\phi = \frac{H'}{a_o^2} \quad ; \quad \tau = \frac{a_o t}{R} \quad ; \quad \zeta = \frac{r}{R} \tag{21}$$

where a_o is the acoustic velocity at stagnation conditions and R is the fictitious radius at which M = 1 for conical flow. In terms of these new variables, the "propagation equation" Eq. (19) becomes

$$\frac{D}{D\tau}\left[\left(\frac{a_o}{a}\right)^2 \frac{D\phi}{D\tau}\right] = \frac{q}{a_o} \cdot \frac{\partial}{\partial \zeta}\left[\left(\frac{a_o}{q}\right)\frac{\partial \phi}{\partial \zeta}\right] \tag{22}$$

where

$$\frac{D}{D\tau} = \frac{\partial}{\partial \tau} \pm \frac{q}{a_o}\frac{\partial}{\partial \zeta} \tag{23}$$

Elrod [1] considered sinusoidal disturbances and found it beneficial to perform a frequency analysis by representing ϕ in the form:

$$\phi = F(\zeta)e^{iW\tau} \tag{24}$$

W is a dimensionless frequency number.

It was also assumed that

$$F(\zeta) = \left(\frac{\alpha}{\beta}\right)^{\frac{1}{2}} \exp\left\{\pm iW\int M\, d\xi\right\}\eta(\xi) \tag{25}$$

The resultant differential equation for $\eta(\xi)$ is

$$\eta''(\xi) + \left[W^2 - g(\xi)\right]\eta(\xi) = 0 \tag{26}$$

where

$$\alpha = \frac{q}{a_0} \qquad ; \qquad \beta = \frac{a_0}{a} \tag{27}$$

$$\xi = \int \frac{\beta}{1-M^2}\, d\zeta \tag{28}$$

and

$$\begin{aligned} g(\xi) &= -\frac{d^2}{d\xi^2} \ln\left(\frac{\alpha}{\beta}\right)^{\frac{1}{2}} + \left\{\frac{d}{d\xi} \ln\left(\frac{\alpha}{\beta}\right)^{\frac{1}{2}}\right\}^2 \\ &= \frac{M^2}{(\zeta\beta)^2}\left[\frac{5\gamma-3}{2} + \frac{\gamma-1}{2}\ \frac{3\gamma-5}{2} M^2\right] \end{aligned} \tag{29}$$

Using the steady state, one-dimensional, isentropic relationships for area and temperature as functions of Mach number (see Shapiro [5]), the following relationships hold

$$\beta = \frac{a_0}{a} = \left(\frac{T_0}{T}\right)^{\frac{1}{2}} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{1}{2}} \tag{30}$$

$$\zeta = \frac{r}{R} = \left(\frac{A}{A^*}\right)^{\frac{1}{2}} = M^{-\frac{1}{2}}\left[\frac{2}{\gamma+1}\ \left(1 + \frac{\gamma-1}{2} M^2\right)\right]^{\frac{\gamma+1}{4(\gamma-1)}} \tag{31}$$

Chiang [2] arrived at the general solution for Eq. (18) by using the WKB method. (See Morse and Feshback [6]).

$$\eta(\xi) = C_1 \exp\left[-iW\xi + \frac{i}{2W}\int g(\xi)\,d\xi\right]$$

$$+ C_2 \exp\left[iW\xi - \frac{i}{2W}\int g(\xi)\,d\xi\right] \tag{32}$$

C_1 and C_2 are arbitrary constants. Eq. (24) now is written as

$$\phi = \left(\frac{\alpha}{\beta}\right)^{\frac{1}{2}} \exp\left[\pm iW\int M\,d\xi\right]\Big\{C_1 \exp\left[iW(\tau-\xi)\right]$$
$$\times \exp\left[\frac{i}{2W}\int g(\xi)\,d\xi\right] + C_2 \exp\left[iW(\tau+\xi)\right]$$
$$\times \exp\left[-\frac{i}{2W}\int g(\xi)\,d\xi\right]\Big\} \tag{33}$$

Chiang [2] states that the C_1 term with the exp $\left[iW(\tau-\xi)\right]$ indicates an outgoing wave (in the positive r-direction) while the C_2 term with exp $\left[iW(\tau+\xi)\right]$ indicates an incoming wave; the term exp $\left[iW\int Md\xi\right]$ represents a mean convection; and the factors exp $\left[\mp\frac{i}{2W}\int g(\xi)d\xi\right]$ represent attenuation of waves. For the case of an infinite conical nozzle, with pressure fluctuations impressed at the throat, $C_2 = 0$.

The WKB solution is asymptotically valid in the limit of small Mach number M and high frequency number W with an error of the order (M^3/W^2). Chiang [2] extended the solution to larger Mach number and smaller frequency number W. A summary of this extension follows:

Assume

$$\eta = c_1 e^{-iw\xi} Z_1(\xi) + c_2 e^{iw\xi} Z_2(\xi) \tag{34}$$

After substituting into the previous differential equations for η and making comparisons, asymptotic approximations of $Z_1(\xi_0)$ and $Z_2(\xi_0)$ are determined.

$$\begin{aligned} Z_1(\xi_0) &= \exp\left[\frac{i}{2W}\int_{\infty}^{\xi_0} g(\xi')\,d\xi'\right] \\ &\cong 1 + \frac{i}{2W}\int_{\infty}^{\xi_0} g(\xi')\,d\xi' \end{aligned} \tag{35}$$

$$\begin{aligned} Z_2(\xi_0) &= \exp\left[-\frac{i}{2W}\int_{\infty}^{\xi_0} g(\xi')\,d\xi'\right] \\ &\cong 1 - \frac{i}{2W}\int_{\infty}^{\xi_0} g(\xi')\,d\xi' \end{aligned} \tag{36}$$

where ξ_0 is sufficiently large (i.e., far enough away from the throat) so that the WKB method holds. Differentiating Eqs. (35) and (36) yields

$$Z_1'(\xi_0) = \frac{1}{2W}\, g(\xi_0) \tag{37}$$

$$Z_2'(\xi_0) = -\frac{1}{2W}\, g(\xi_0) \tag{38}$$

It is further shown that Z_1 and Z_2 are complex conjugates and that Eqs. (35) - (38) can be used for initial conditions to solve the differential equations for Z_1 and Z_2.

Let

$$\left.\begin{aligned} Z_1 &= u_1 + i\,v_1 \\ Z_2 &= u_2 + i\,v_2 \end{aligned}\right] \tag{39}$$

then

$$u_1 = u_2 \qquad , \qquad v_1 = v_2 \tag{40}$$

After changing variables by using

$$x = M^2 \tag{41}$$

and

$$\Phi = \frac{dx}{d\xi} \tag{42}$$

Chiang showed that

$$\Phi = -4\frac{\beta x}{\xi} \tag{43}$$

and arrived at the following differential equations

$$u_1''(x) + \frac{1}{\Phi}\left[\frac{d\Phi}{dx}\,u_1'(x) + 2W\,v_1'(x)\right] - \frac{g(x)}{\Phi^2}\,u_1(x) = 0 \tag{44}$$

$$v_1''(x) + \frac{1}{\Phi}\left[\frac{d\Phi}{dx}\,v_1'(x) - 2W\,u_1'(x)\right] - \frac{g(x)}{\Phi^2}\,v_1(x) = 0 \tag{45}$$

with the initial conditions

$$u_1(x_0) = 1$$

$$u_1'(x_0) = 0 \tag{46}$$

$$v_1(x_0) = \frac{1}{2W} \int_0^{x_0} \frac{g(x)}{\Phi} \, dx$$

$$v_1'(x_0) = \frac{1}{2W} \quad \frac{g(x_0)}{\Phi(x_0)}$$

where x_0 is the value of x corresponding to $\xi = \xi_0$. β, ζ, Φ, and g have all been previously defined as functions of M and are therefore functions of x.

It is always possible to select a Mach number M small enough to satisfy the restriction on the WKB solution,

$$\frac{M^3}{W^2} << 1 \tag{47}$$

The amplitude and phase relations between pressure and mass flux perturbations are determined in the following manner. Eqs. (8) and (10) are used to obtain

$$\frac{\partial q'}{\partial t} = \mp \frac{\partial H'}{\partial r} \tag{48}$$

$$\frac{1}{\rho} \frac{\partial p'}{\partial t} = \frac{\partial H'}{\partial t} \pm q \frac{\partial H'}{\partial r} \tag{49}$$

The mass flux perturbation is written

$$(\rho q)' = \rho q' + q \rho' \tag{50}$$

After introducing the dimensionless pressure and mass flux perturbations:

$$\pi = \frac{p'}{p} \tag{51}$$

$$\mu = \frac{(\rho q)'}{\rho q} \tag{52}$$

Chiang [2] showed that

$$\frac{\dfrac{\partial \pi}{\partial \tau}}{\dfrac{\partial \mu}{\partial \tau}} = \gamma M \frac{\dfrac{\partial \ln \phi}{\partial \tau} \pm \dfrac{M}{1-M^2} \dfrac{\partial \ln \phi}{\partial \xi}}{M \dfrac{\partial \ln \phi}{\partial \tau} \mp \dfrac{\partial \ln \phi}{\partial \xi}} \tag{53}$$

Furthermore, since both π and μ are periodic in time and contain the factor $\exp\left[i\omega\tau\right]$

$$\frac{\pi}{\mu} = \frac{\dfrac{\partial \pi}{\partial \tau}}{\dfrac{\partial \mu}{\partial \tau}} \tag{54}$$

For the case of an infinite conical nozzle with sinusoidal pressure fluctuations impressed at the throat, C_2 in Eq. (34) becomes zero and

$$\eta(\xi) = C_1 \exp\left[-iW\xi\right] (u_1 + i v_1) \tag{55}$$

therefore

$$\phi = C_1 \left(\frac{\alpha}{\beta}\right)^{\frac{1}{2}} \exp\left[-iw \int M \, d\xi\right] \times \exp\left[iw(\tau - \xi)\right] (u_1 + i v_1) \tag{56}$$

Taking the logarithm of Φ and differentiating gives

$$\frac{\partial \ln \phi}{\partial \tau} = iW \tag{57}$$

$$\frac{\partial \ln \phi}{\partial \xi} = \left\{ \frac{\Phi}{4}\left(\frac{1}{x} - \frac{\gamma - 1}{\beta^2}\right) + \frac{\Phi}{u_1^2 + v_1^2}\left(u_1 \frac{du_1}{dx} + v_1 \frac{dv_1}{dx}\right)\right\} + i\left\{-W(x^{\frac{1}{2}} + 1) + \frac{\Phi}{u_1^2 + v_1^2}\left(u_1 \frac{dv_1}{dx} - v_1 \frac{du_1}{dx}\right)\right\} \tag{58}$$

Substitution of Eq. (57) and (58) into Eq. (53), together with Eq. (54) yields

$$\frac{\Pi}{\mu} = \frac{\frac{\partial \Pi}{\partial \tau}}{\frac{\partial \mu}{\partial \tau}} = \frac{\gamma M^2}{1 - M^2} \bar{N} e^{i\bar{\Psi}} \tag{59}$$

where

$$\bar{N} \quad = \text{amplitude factor} = \left(\frac{N_1^2 + N_2^2}{N_3^2 + N_4^2}\right)^{\frac{1}{2}} \tag{60}$$

$$\bar{\Psi} \quad = \text{phase angle} = \arctan\left(\frac{N_2 N_3 - N_1 N_4}{N_1 N_3 + N_2 N_4}\right) \tag{61}$$

and

$$N_1 = \frac{\Phi}{4}\left(\frac{1}{x} - \frac{\gamma-1}{\beta^2}\right) + \frac{\Phi}{u_1^2+v_1^2}\left(u_1\frac{du_1}{dx} + v_1\frac{dv_1}{dx}\right)$$

$$N_2 = -W\left(x^{-\frac{1}{2}}+1\right) + \frac{\Phi}{u_1^2+v_1^2}\left(u_1\frac{dv_1}{dx} - v_1\frac{du_1}{dx}\right) \tag{62}$$

$$N_3 = -N_1$$

$$N_4 = -N_2 - W x^{-\frac{1}{2}}$$

A dimensionless parameter designated as the modified Strouhal number S* was defined by Chiang [2]

$$S^* = \frac{\omega r}{q} \tag{63}$$

and shown to be related to the frequency number

$$W = \frac{M S^*}{\beta q} \tag{64}$$

For a given Mach number M, it is more convenient to express the mass flux-pressure relationships in terms of the modified Strouhal number S* rather than the frequency number W.

III. NUMERICAL SOLUTION

Equations (44) and (45) are transformed to a system of four first order differential equations by defining the variables:

$$
\begin{aligned}
y_1(x) &= u_1(x) \\
y_2(x) &= u_1'(x) \\
y_3(x) &= v_1(x) \\
y_4(x) &= v_1'(x)
\end{aligned}
\tag{65}
$$

The resultant system of differential equations is

$$
\begin{aligned}
y_1' &= y_2 \\
y_2' &= -\frac{1}{\Phi}\left[\frac{d\Phi}{dx} y_2 + 2W y_4\right] + \frac{g(x)}{\Phi^2} y_1 \\
y_3' &= y_4 \\
y_4' &= -\frac{1}{\Phi}\left[\frac{d\Phi}{dx} y_4 - 2W y_2\right] + \frac{g(x)}{\Phi} y_3
\end{aligned}
\tag{66}
$$

A numerical solution for this system of equations was obtained using Hamming's Modified Predictor-Corrector Method of numerical integration. (See Ralston and Wilfe [7]). It is a stable, fourth order integration technique which uses four preceding points for the evaluation of each successive point. PROGRAM TWO makes use of an IBM [8] subroutine, DHPCG, to perform the numerical integration.

Evaluation of the initial conditions $v_1(x_o)$ and $v_1(x_o)$ is accomplished using PROGRAM ONE. Subroutine GOPHI evaluates the function $g(x)/\Phi$. Numerical integration of $v_1(x_o)$ is performed using a twelve

point Gaussian Quadrature formula which integrates polynomials up to degree 23 exactly. Subroutine DQG12 is used for this purpose. (See IBM [8], p. 299.)

With u_1, u_1, v_1, and v_1 known, subroutine OUTP of PROGRAM TWO computes values of amplitude Factor $\overline{N}$ and phase angle $\overline{\psi}$ for a given Mach number M and frequency number W. The corresponding modified Strouhal number S* is also computed.

The programs have been written for the case of air with $\gamma = 1.4$.

IV. EXPERIMENTATION

A. EXPERIMENTAL APPARATUS

To approximate the characteristics of an infinite conical nozzle, a nozzle 17.5 inches in length with an 8 inch diameter entrance and a 1 inch diameter exit was designed and constructed. It subtended a half-cone angle of 11.3 degrees and featured an entrance-exit planar area ratio of 64. This ensured that stagnation conditions existed at the entrance for all subsonic flows in the nozzle. (See Shaprio [5], p. 614.)

Figures 1 and 2 show the overall arrangement of the experimental apparatus and associated electronic equipment. Figure 3 schematically depicts the test section.

Compressed air from two supply tanks passed through a pressure regulating valve and into the 10 inch diameter stagnation chamber through a baffling arrangement. The stagnation chamber contained honeycomb material with a length/diameter ratio of 10 to dampen out any disturbances in the plenum air. A fine mesh screen was also installed to hasten turbulence decay prior to the entrance of the flow into the nozzle.

Sinusoidal pressure disturbances in the nozzle exit plane were created by a vibrating piston located in the exhaust chamber. The piston was driven by a MB Electronics Model EA 1250 Vibramatic Exciter powered by a Model 2120 Amplifier using a sinusoidal signal from a Hewlett-Packard Model 200CD Wide Range Oscillator. The vibrator was mounted on three horizontal tracks so that the zero position of the piston could be adjusted for optimum operation.

Dynamic pressure measurements in the exit plane were recorded with a Bruel and Kjaer Model 4138 1/8 inch Condenser Microphone with Adaptor UA0036 and Model 2618 1/4 inch Preamplifier connected to a Model 2803 Two-Channel Power Supply.

Mass flux (ρq) measurements were performed using a Thermo-Systems, Incorporated Model 1050 Constant Temperature Anemometer with a DISA Type 55F33 Hot Wire Probe. The hot wire anemometer measured both steady state (DC) and dynamic (AC) components of mass flux. The anemometer DC voltage was measured on a Simpson Model 2700 Digital Voltmeter.

The anemometer and microphone signals were passed through identical Krohn-Hite Model 3750 Filters (band-pass) to preserve phase. The AC voltages were measured by two Hewlett-Packard Model 3400A RMS Voltmeters. The filtered dynamic signals were displayed on a Hewlett-Packard Model 132A Dual Beam Oscilloscope for visual verification of system response.

Phase angles between pressure and mass-flux fluctuations were measured on a Dranetz Model 305-PA-3002 Digital Phase Meter using the output signals from the oscilloscope amplifiers. Figure 4 shows a block diagram of the electronic measurement equipment.

Manometers were used to measure stagnation and exit static pressures. Stagnation temperature was measured by a mercury-in-glass thermometer.

B. PROCEDURE

Mass flux (ρq) calibration of the hot wire anemometer was performed using a Thermo-Systems, Incorporated Model 1125 Calibrator. The method of least squares was used to fit an eighth order polynomial in the quantity $(V^2 - V_o^2)$ to the calibration points. (See Computer PROGRAM THREE.) Microphone calibration was performed with a Bruel and Kjaer Piston Phone: Type 4220.

Each experimental run consisted of first setting a steady state flow rate by maintaining a constant pressure in the stagnation chamber. The frequency and amplitude of the pressure disturbances were set on the oscillator. Both band-pass filters were set to pass only a narrow band at the disturbance frequency with gain settings of 20 db. Data was taken over the range of frequencies from 20 Hz to 250 Hz.

C. DATA REDUCTION

Reduction of experimental data was accomplished on a digital computer through the use of PROGRAM FOUR.

Steady state mass flux was computed by using the anemometer DC voltage in the evaluation of the hot wire polynomial. The magnitude of mass flux perturbation was obtained by first correcting the anemometer RMS voltage and then adding it to the DC voltage to yield a peak voltage. The hot wire polynomial was then evaluated for the peak voltage, giving a maximum value of mass flux. The steady state value was then subtracted from the maximum value to arrive at the perturbation.

Microphone RMS voltage was similarly corrected to obtain a peak value and then multiplied by the microphone sensitivity to yield the magnitude of the pressure perturbation. A 6 db. narrow band insertion loss in the

band-pass filters was also accounted for in the computations of dynamic quantities.

Mach number was computed from the one dimensional, isentropic pressure relationship. The velocity q used in the computation of Modified Strouhal number S* was determined by dividing the mass flux by a value of ρ arrived at by using the perfect gas law and the isentropic pressure-density relationship.

V. DISCUSSION OF RESULTS AND CONCLUSIONS

Figures 5 through 14 show a comparison of the experimentally determined amplitude factors and phase angles with values predicted by the Modified WKB solution for Mach numbers: .084, .117, .201, .278, and .340. Both amplitude factor and phase angle are plotted against modified Strouhal number S*. The smooth curves are the modified WKB solutions and the experimental results are represented by crosses.

For low frequency sinusoidal pressure disturbances with corresponding small values of S*, the phase angle is -180 degrees in accordance with the modified WKB solution which asymptotically approaches that value. An increase in pressure results in a decrease in mass flux and physically it is easy to visualize an effective phase angle between the rising pressure and the falling mass flux. Mathematically, this effective phase angle is simply 180° plus the value of the actual phase angle. Thus, for low frequencies there is no effective lag between π and μ.

The WKB values of amplitude factor monotonically increase from unity for low S* to the asymptotic limit for extremely high S*. This means that the magnitude of the mass flux response diminishes as the frequency of the pressure disturbances increases.

Chiang [2] noted that the quasi-steady relationship was probably valid for low values of S* but became increasingly inaccurate for larger values of S*.

The experimental values of amplitude factor agree quite well with those predicted for all Mach numbers tested. The experimental values for phase angle differed from those predicted but exhibited a similar trend. For the low frequency tested (20 Hz) all experimental phase angles were close to -180 degrees while the WKB values were slightly higher. For higher frequencies, the experimental phase angles became larger than the WKB predicted values. This apparent phase shift can be attributed to effects of viscosity in the flow. Viscosity serves to dampen the flow system with a resultant increase in effective phase angle over the undamped system, i.e., that assumed in the WKB analysis.

Physical limitations on the test apparatus prevented investigations in the regions above M = .34 and disturbance frequencies of 250 Hz. Electronic noise and vibrations generated by the air compressor prevented its being run during experiments. Consequently, the air supply in the tanks could not sustain runs over a frequency range at higher Mach numbers. The other problem encountered was that of turbulence. Early investigations revealed that turbulence in the nozzle was prevalent at frequencies below 200 Hz. Inclusion of the fine mesh screen in the stagnation chamber helped greatly to dissipate turbulence or at least to break it up into high frequency turbulence. Power density spectrums of the hot wire anemometer signals, obtained by taking the Fourier Transform of the autocorrelation function, indicated the presence of turbulence with frequencies above 250 Hz. This high frequency turbulence created too high a signal to noise level for meaningful data to be obtained

above 250 Hz. A PAR Model 101A Correlation Function Computer and a Model 102 Fourier Analyzer were used to determine the power density spectrums.

It is concluded that the modified WKB solution, with its ideal flow assumptions, does offer a reasonable approximation of the amplitude factor for pulsating subsonic flow of air in a conical nozzle, as long as the flow originates at near-stagnation conditions. Prediction of phase angle is conservative and could be used as a lower bound.

It is recommended that further investigations be pursued at higher Mach numbers. This would require both a quiet compressed air supply and a higher power vibrational exciter. It is felt that the present location of the air compressor within the laboratory is unsatisfactory and that the compressor should be isolated from the laboratory by either moving it outside of the building or by some other suitable means.

It is also recommended that an analysis be undertaken which would account for the effects of viscosity upon the dynamics of a pulsating flow field in a convergent nozzle.

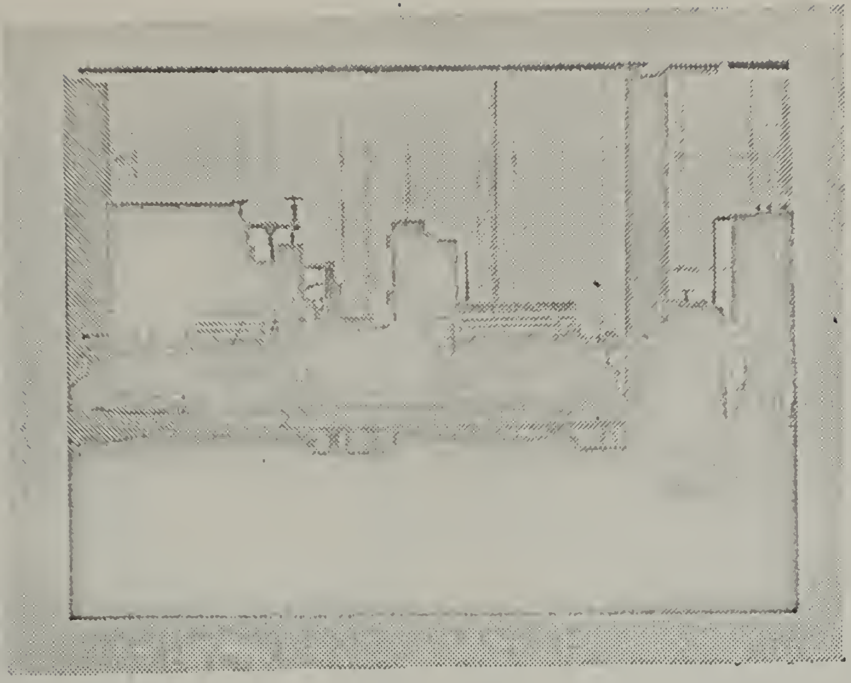

Figure 1. TEST APPARATUS

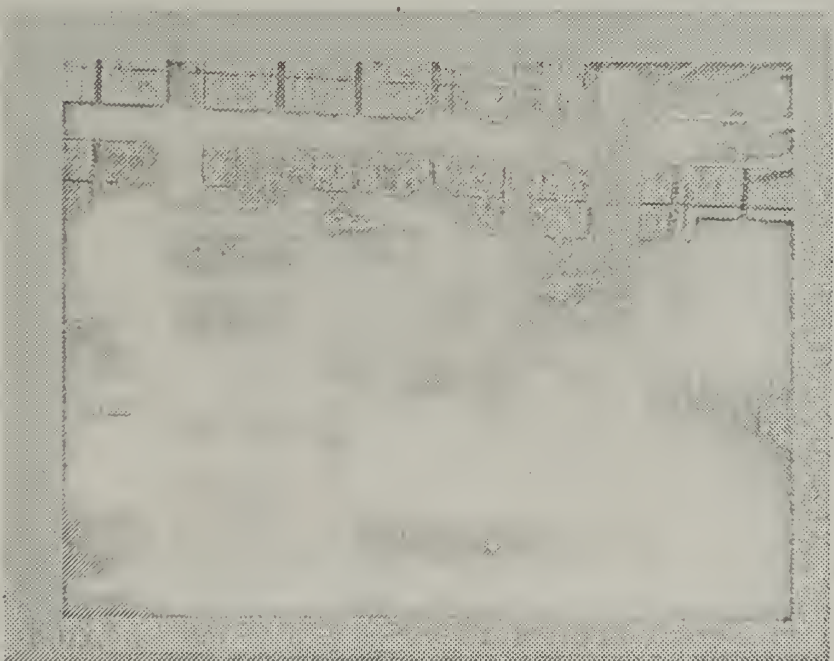

Figure 2. ELECTRONIC EQUIPMENT

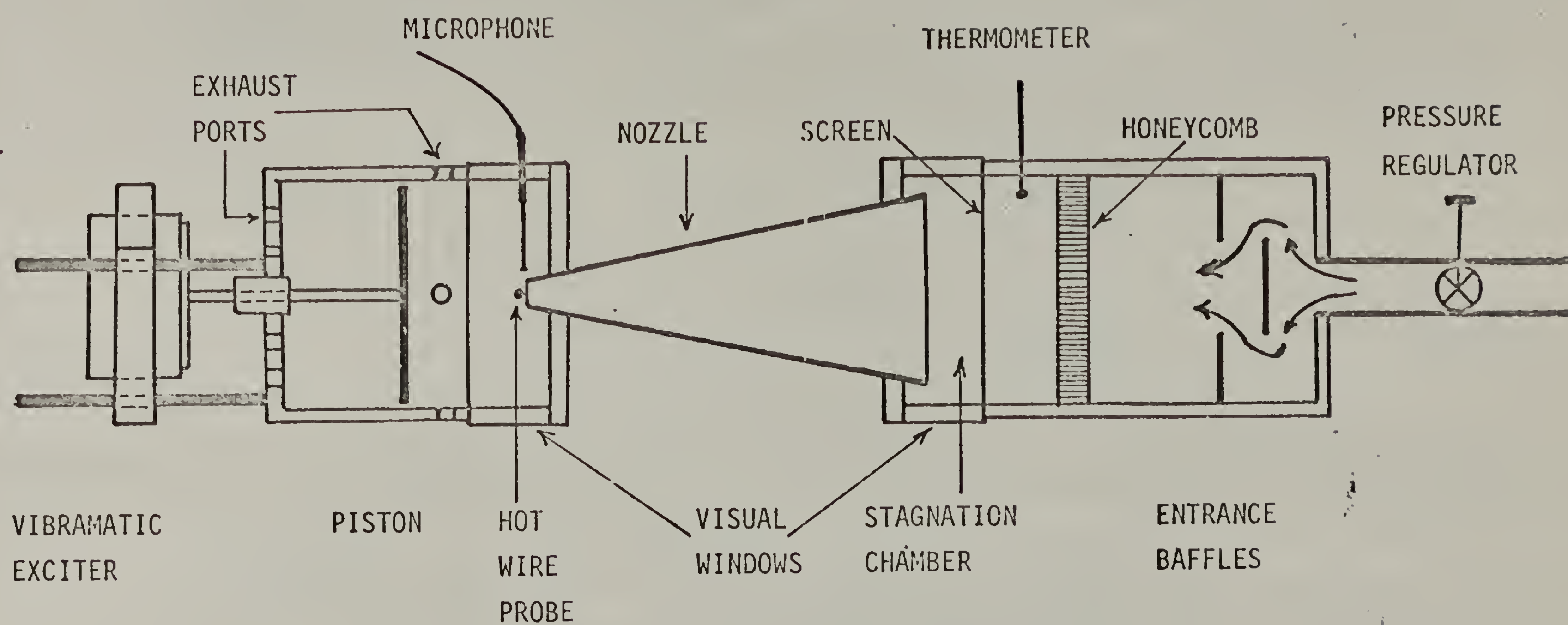

FIGURE 3. SCHEMATIC OF TEST APPARATUS

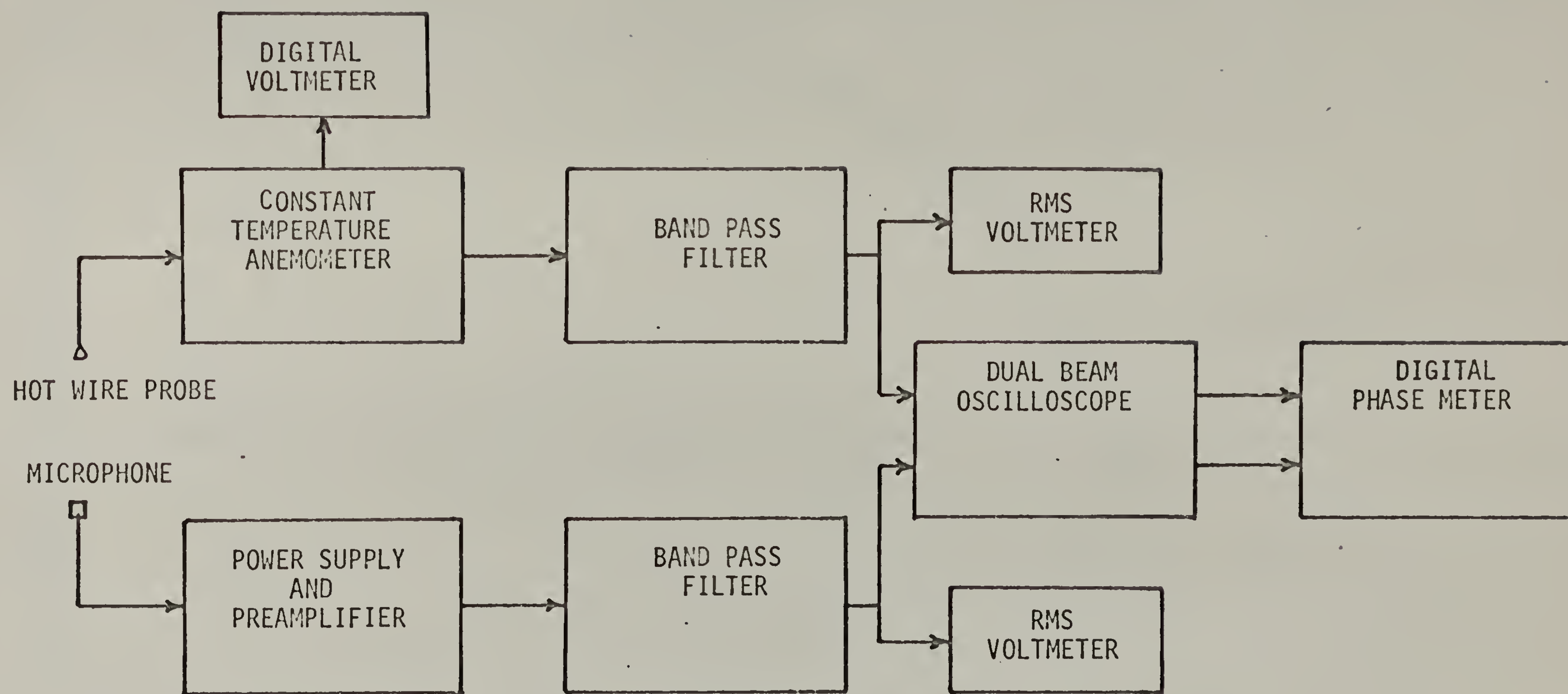

FIGURE 4. BLOCK DIAGRAM OF ELECTRONIC MEASUREMENT EQUIPMENT

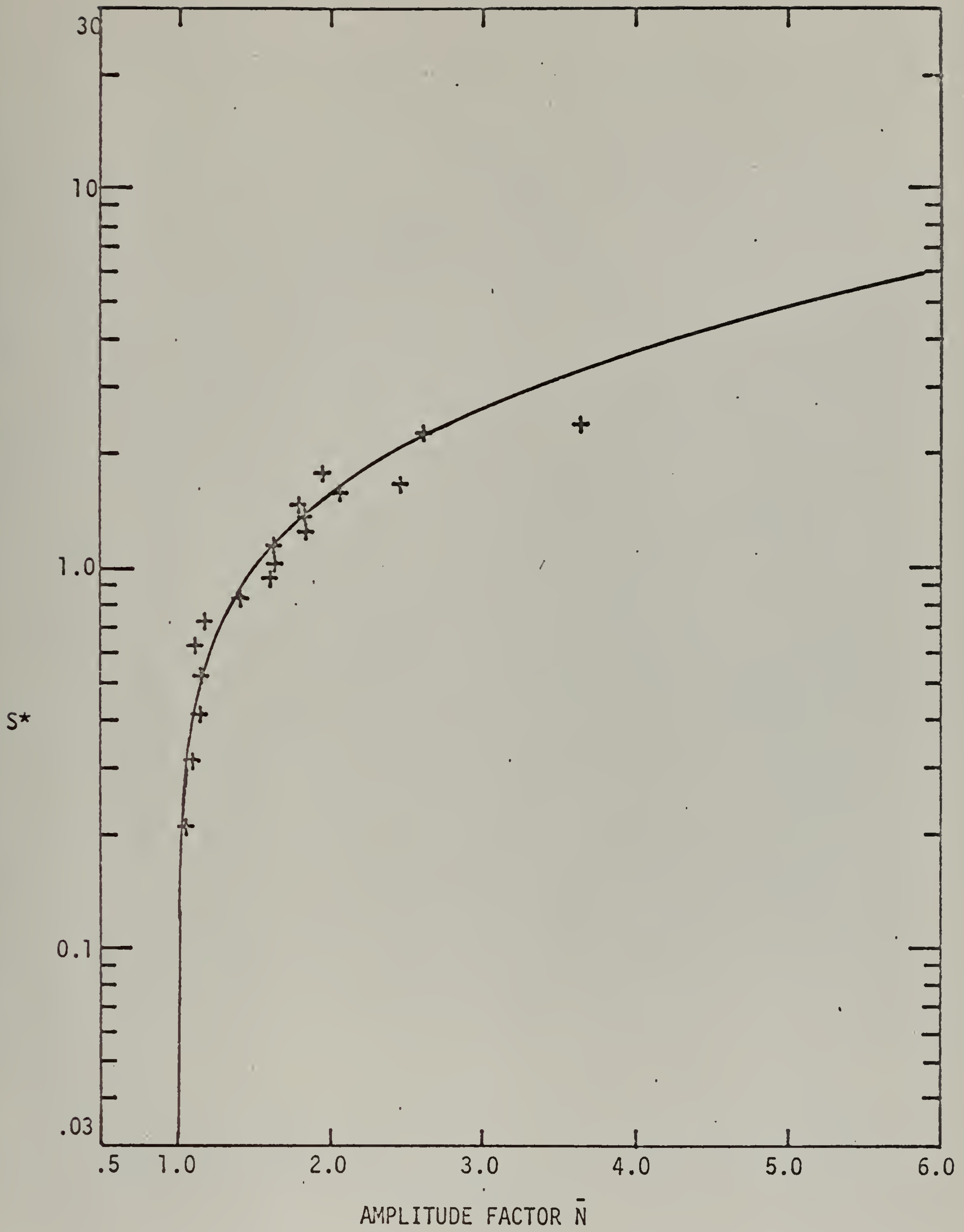

FIGURE 5. COMPARISION OF EXPERIMENTAL AMPLITUDE FACTOR WITH MODIFIED WKB SOLUTION, M = .084

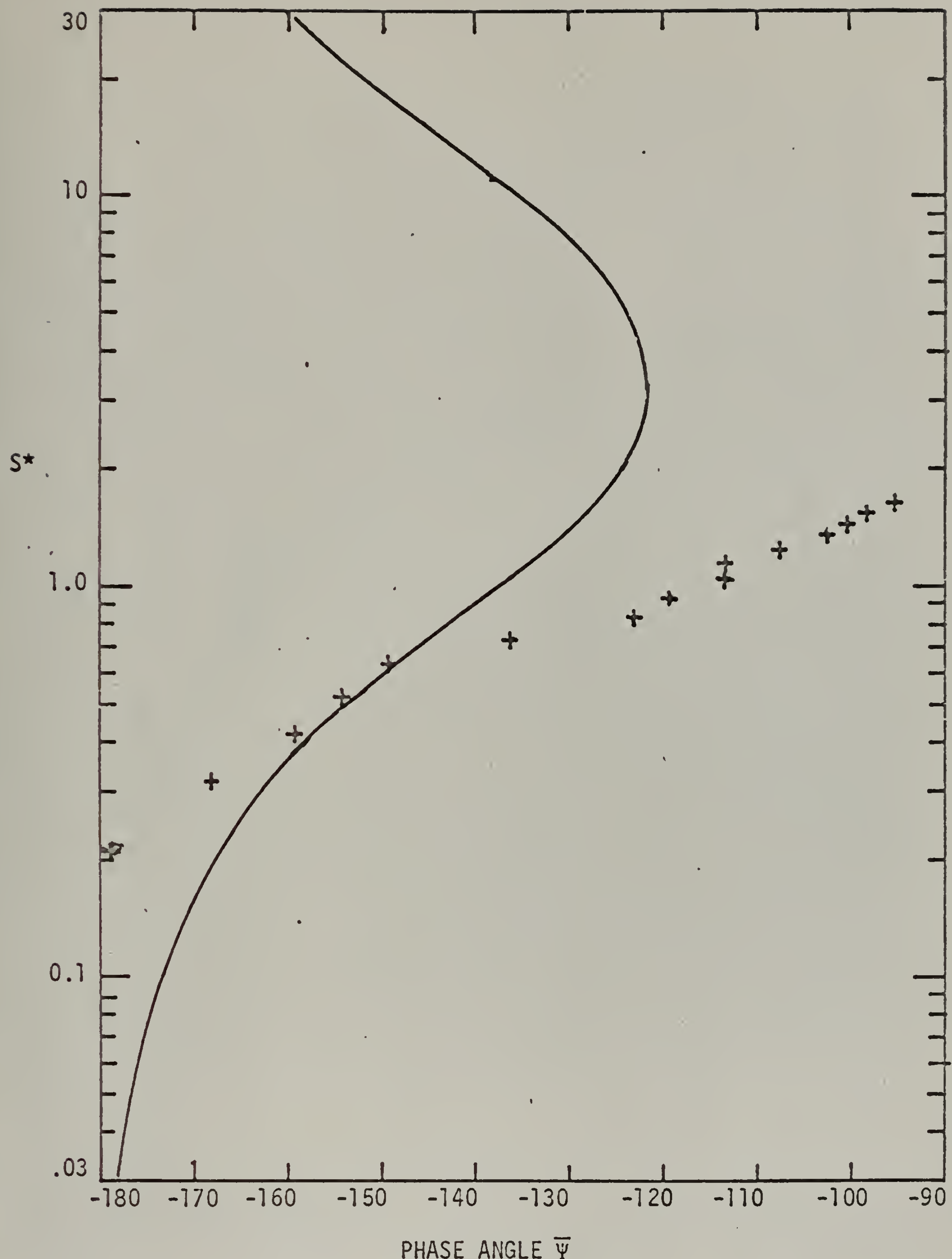

FIGURE 6. COMPARISON OF EXPERIMENTAL PHASE ANGLE WITH MODIFIED WKB SOLUTION, M = .084

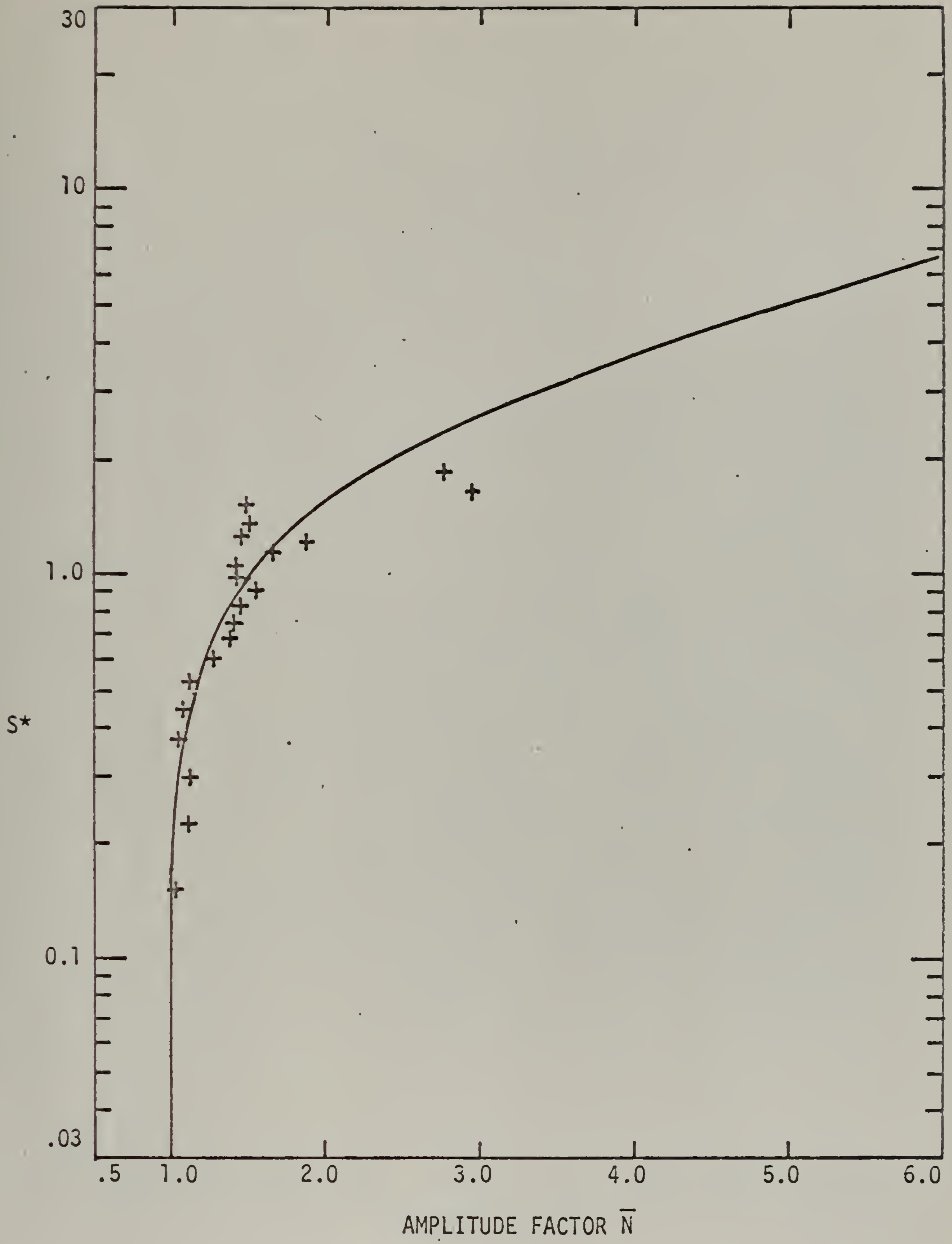

FIGURE 7. COMPARISON OF EXPERIMENTAL AMPLITUDE FACTOR WITH MODIFIED WKB SOLUTION, M = .117

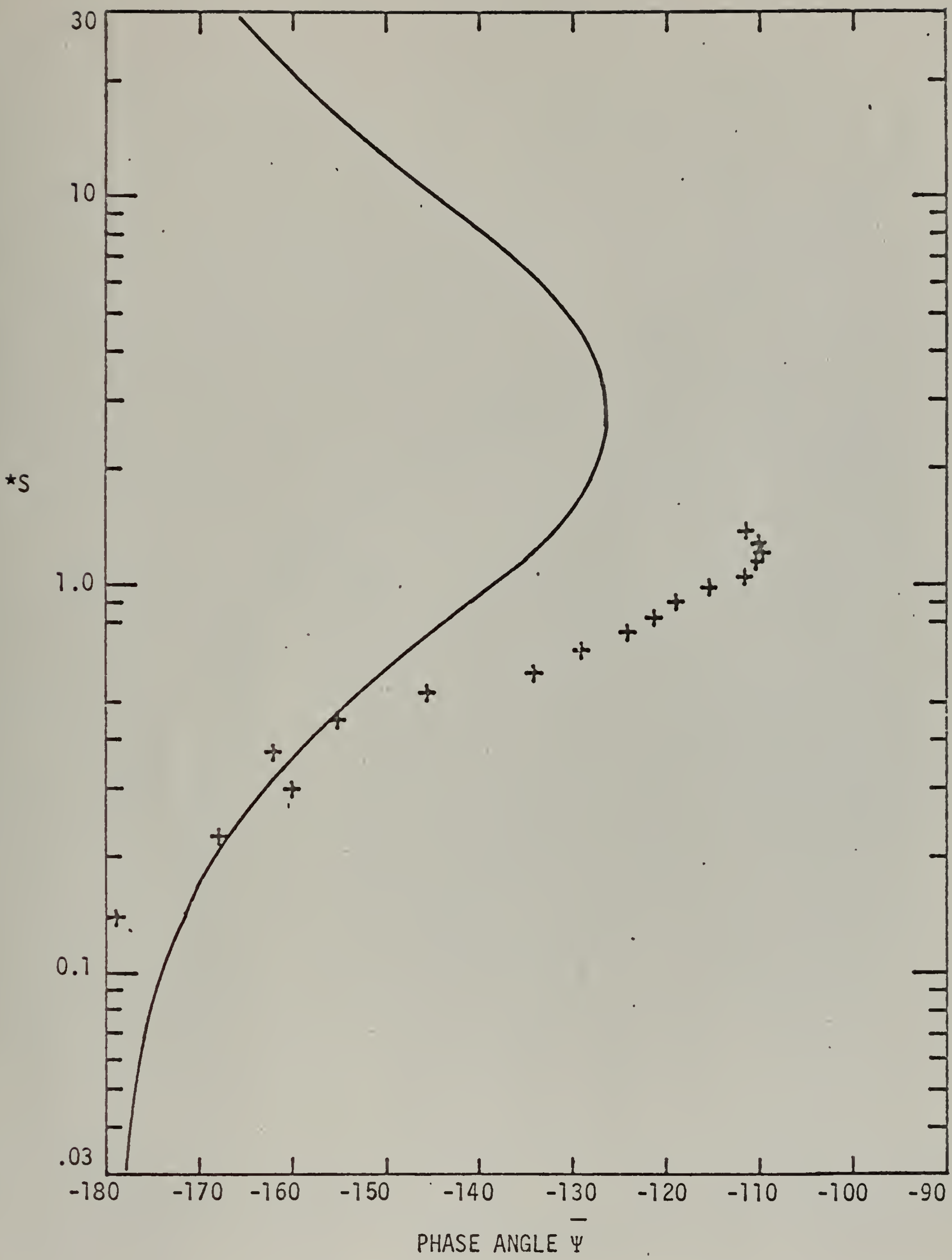

FIGURE 8. COMPARISON OF EXPERIMENTAL PHASE ANGLE WITH MODIFIED WKB SOLUTION, M = .117

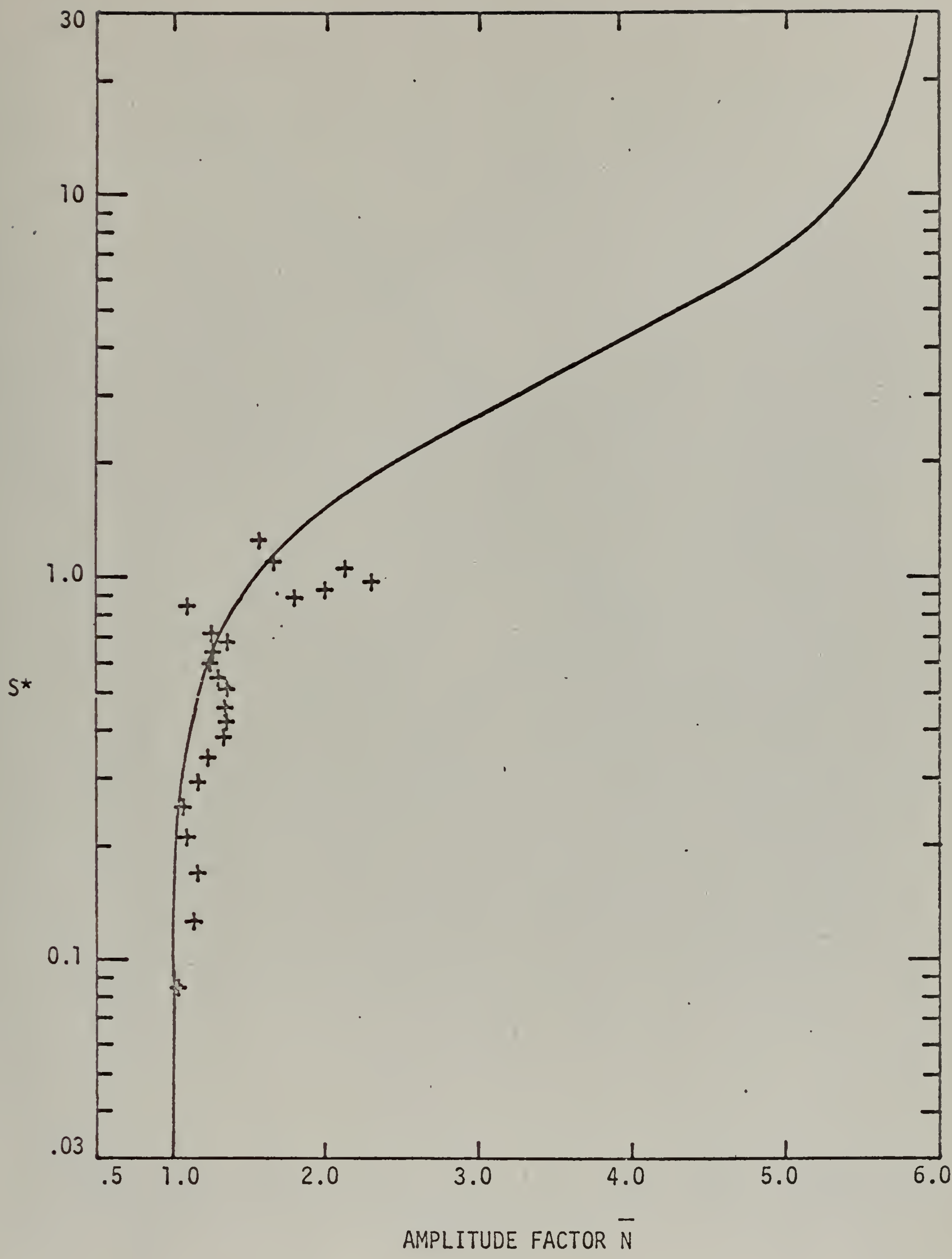

FIGURE 9. COMPARISON OF EXPERIMENTAL AMPLITUDE FACTOR WITH MODIFIED WKB SOLUTION, M = .201

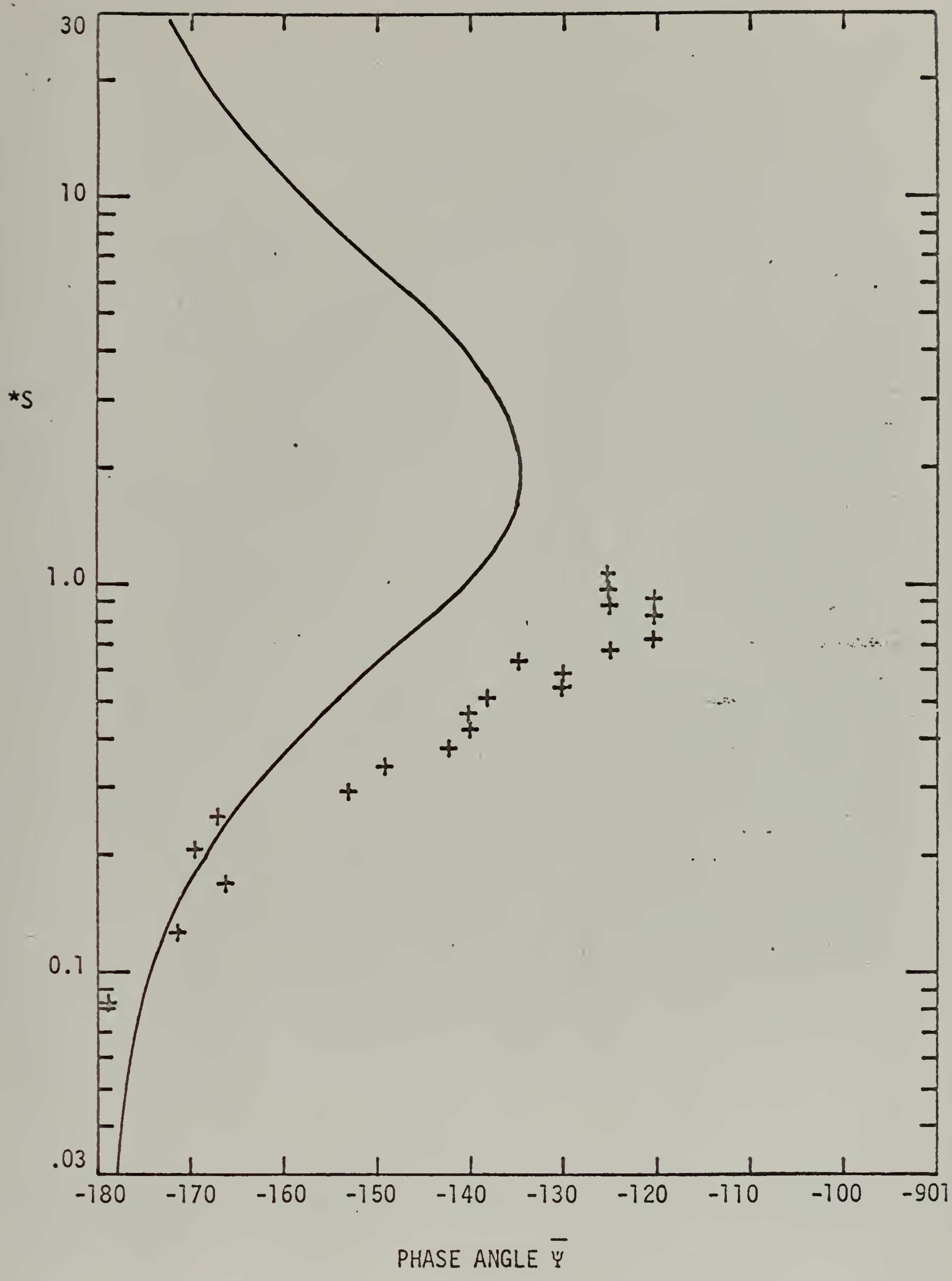

FIGURE 10. COMPARISON OF EXPERIMENTAL PHASE ANGLE WITH MODIFIED WKB SOLUTION, M = .201

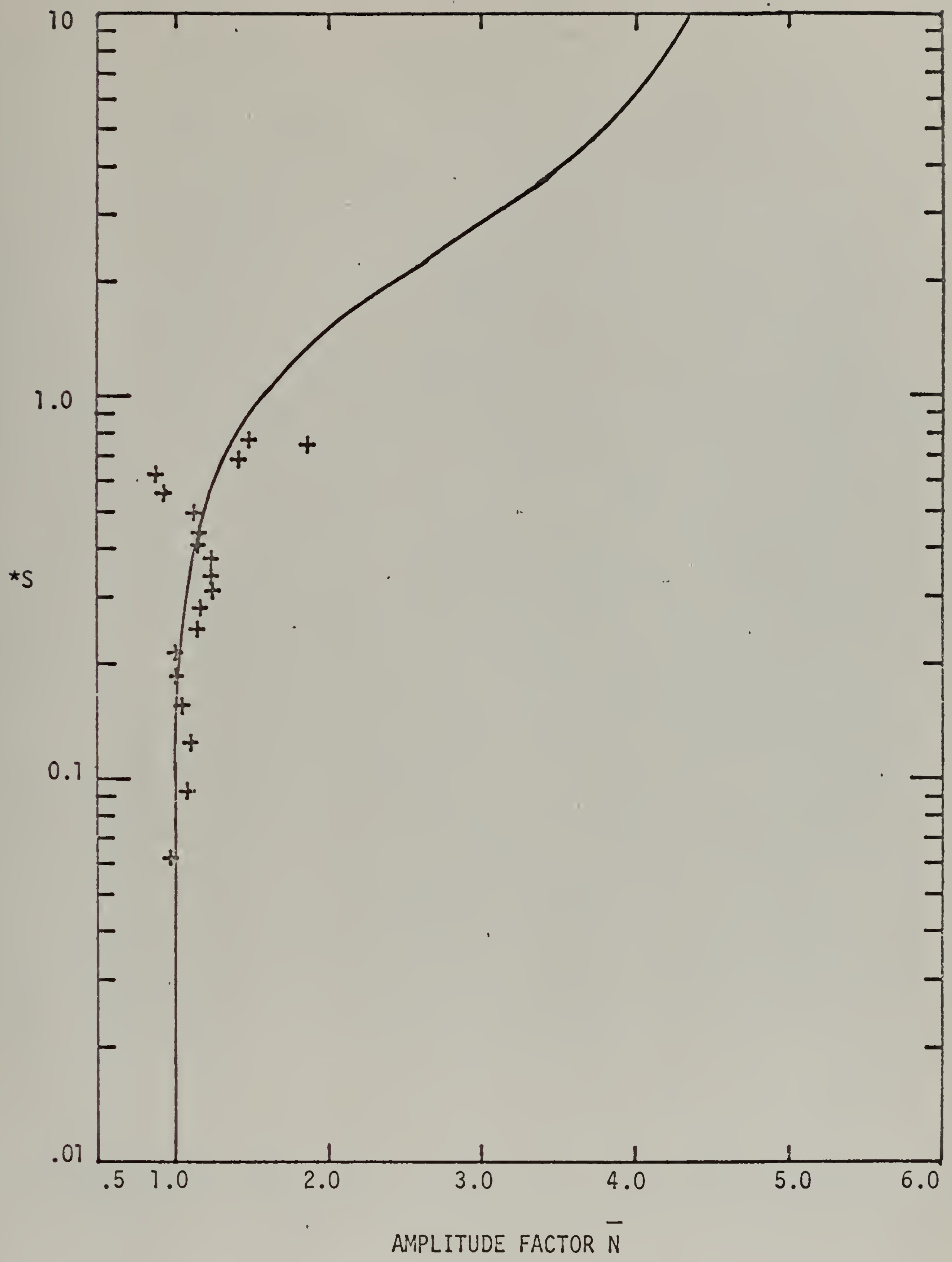

FIGURE 11. COMPARISON OF EXPERIMENTAL AMPLITUDE FACTOR WITH MODIFIED WKB SOLUTION, M = .278

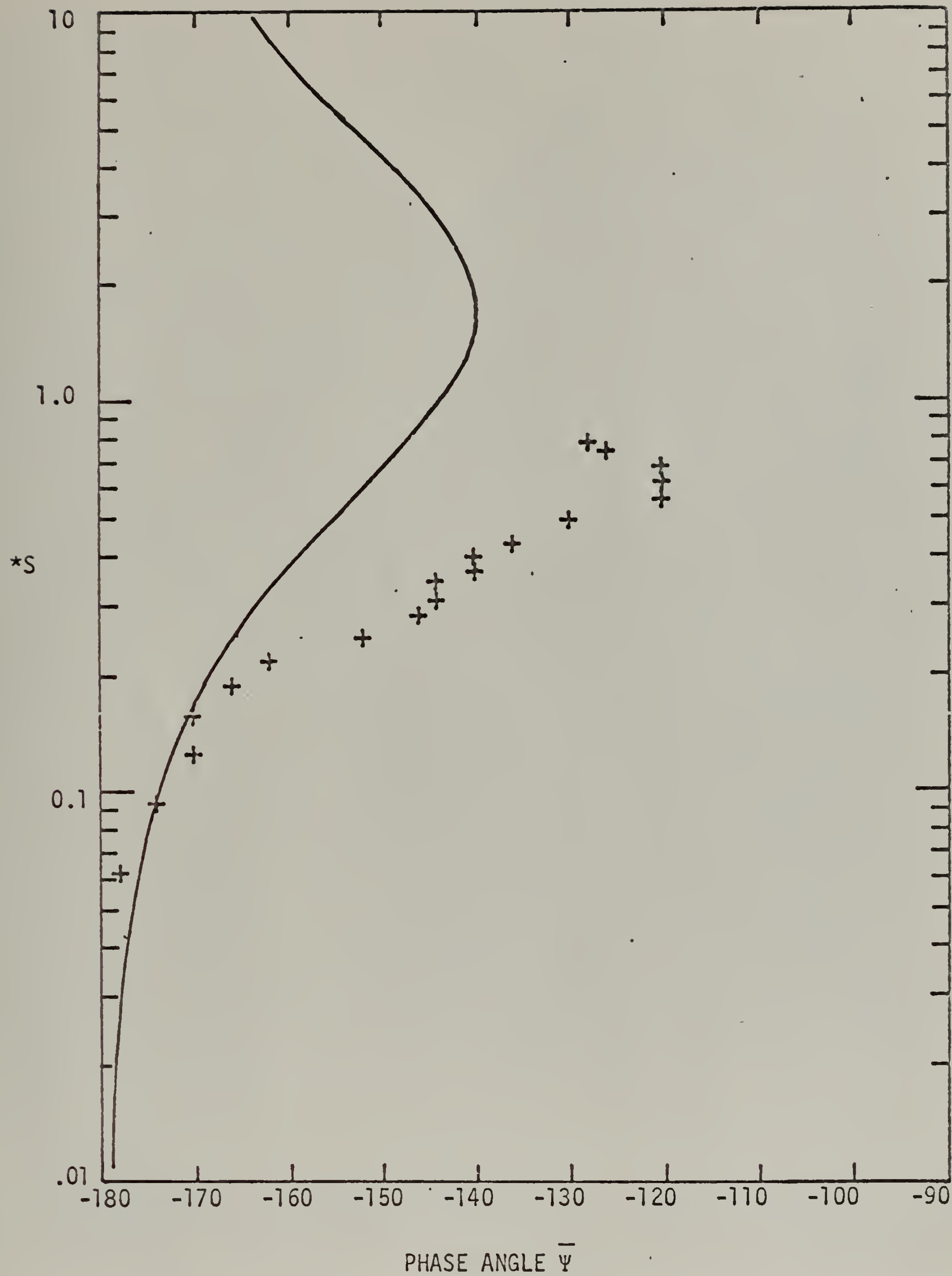

FIGURE 12. COMPARISION OF EXPERIMENTAL PHASE ANGLE WITH MODIFIED WKB SOLUTION, M = .278

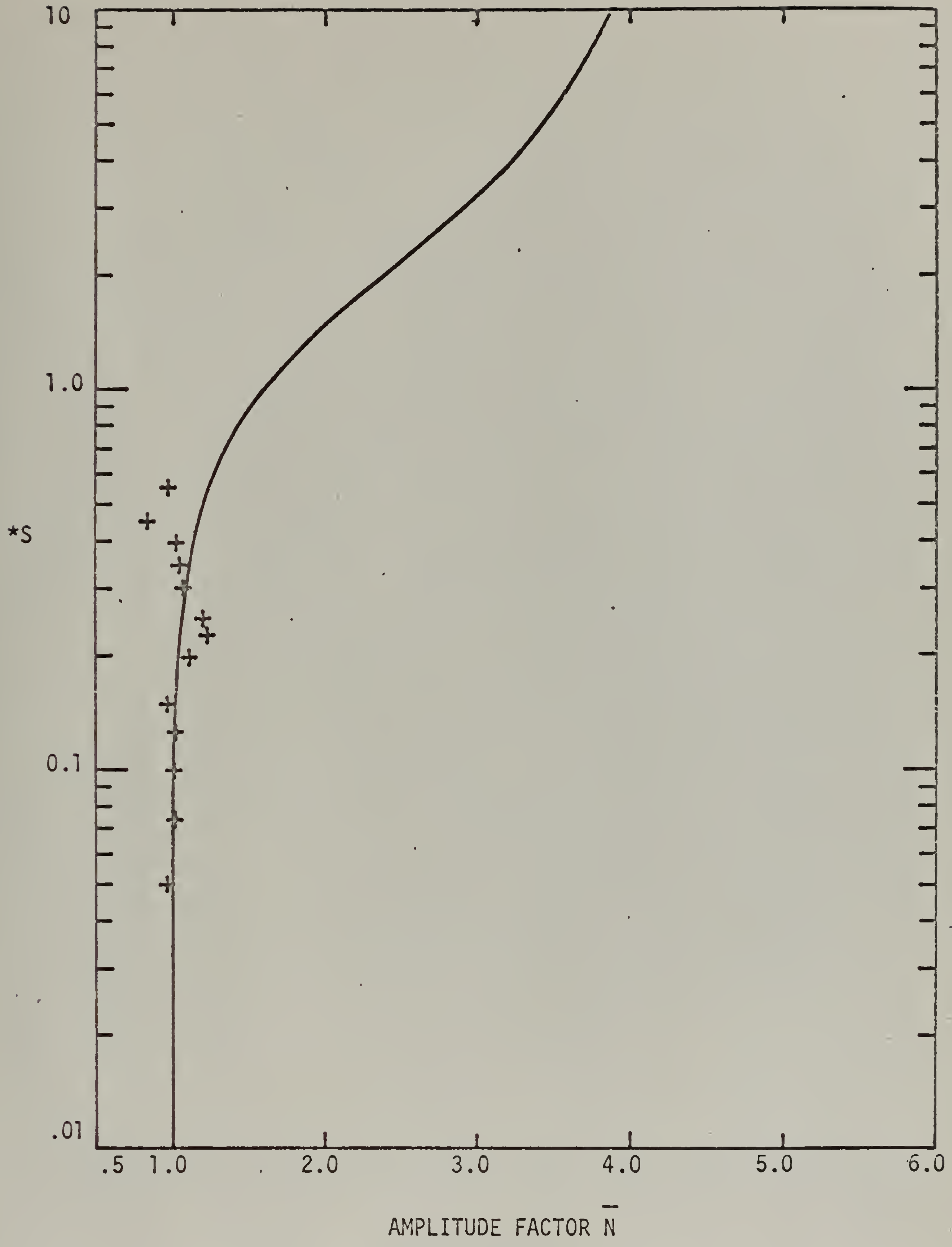

FIGURE 13. COMPARISON OF EXPERIMENTAL AMPLITUDE FACTOR WITH MODIFIED WKB SOLUTION, M = .340

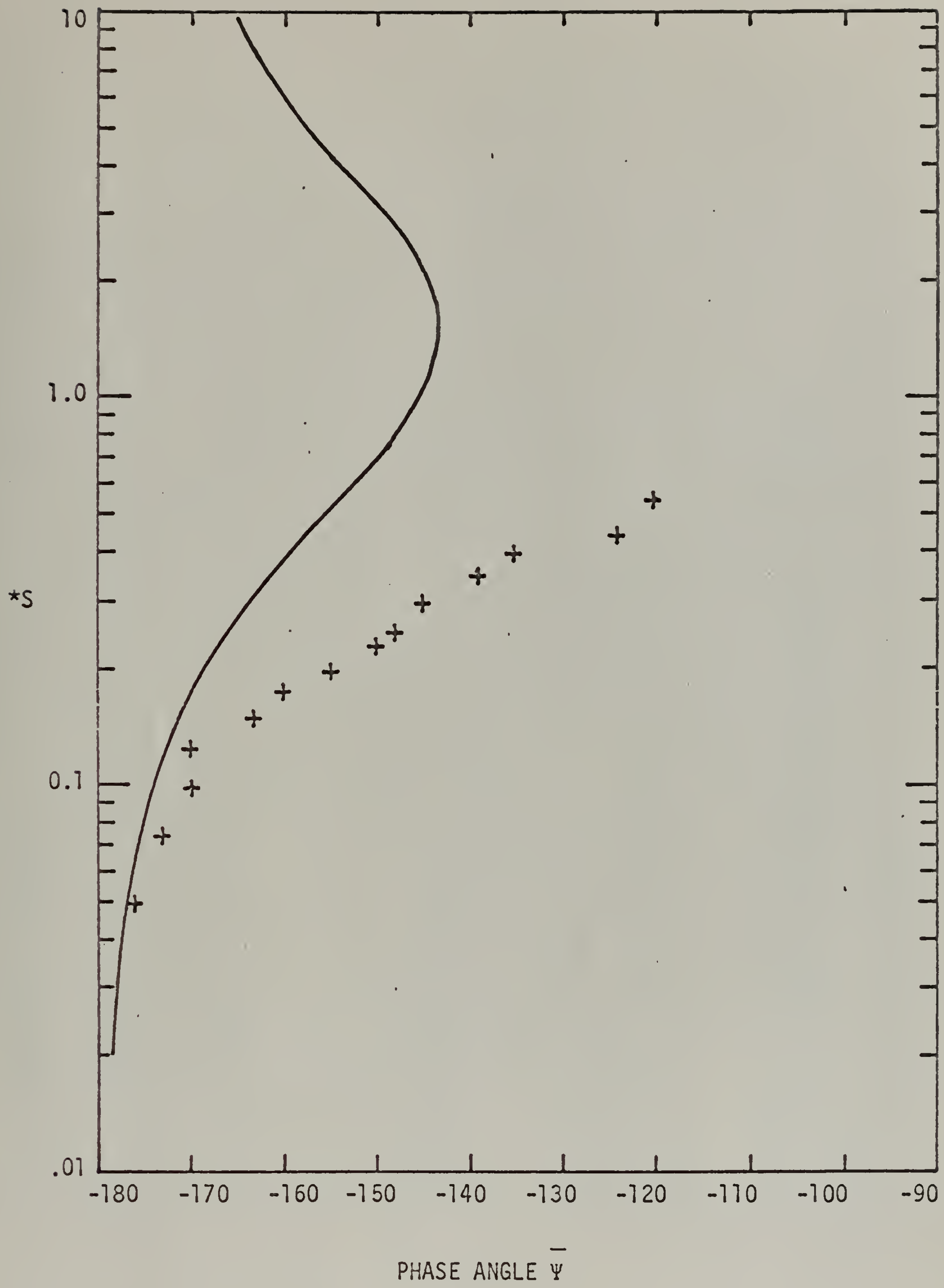

FIGURE 14. COMPARISON OF EXPERIMENTAL PHASE ANGLE WITH MODIFIED WKB SOLUTION, M = .340

```
C              COMPUTER PROGRAMS
C
C
C
C     ************************************************************
C              PROGRAM  ONE
C     ************************************************************
C
C     EVALUATION OF INITIAL CONDITIONS V1(X0), V1'(X0)
C
      IMPLICIT REAL*8 (G,V,D,W)
      EXTERNAL GOPHI
C     W = FREQUENCY NUMBER
      W=1.D0
      X0 = .0001
C
C     COMPUTE V1(X0)
      CALL DQG12(0.D0,1.D-4,GOPHI,V)
      V=V/W/2.D0
      WRITE(6,100) V
  100 FORMAT('0',D25.17)
C     COMPUTE V1'(X0)
      DV=GOPHI(1.D-4)/W/2.D0
      WRITE(6,100)DV
      STOP
      END


      FUNCTION GOPHI(X)
C     EVALUATION OF G(X)/PHI(X)
      IMPLICIT REAL*8(A,B,Z,G,X)
      A=2.D0-.08D0*X
      BETA3=(1.D0+.2D0*X)**1.5D0
      ZETAIN=(X**.25D0)/((1.D0+.2D0*X)/1.2D0)**1.5D0
      GOPHI=-A*ZETAIN/BETA3/4.D0
      RETURN
      END


      SUBROUTINE DQG12(XL,XU,FCT,Y)
C     NUMERICAL INTEGRATION USING 12 PT. GAUSSIAN QUADRATURE
C     Y = THE INTEGRAL OF FCT OVER THE INTERVAL ( XL TO XU )
      DOUBLE PRECISION XL,XU,Y,A,B,C,FCT
      A=.5D0*(XU+XL)
      B=XU-XL
      C=.4907803171233596300*B
      Y=.2358766819325591 4D-1*(FCT(A+C)+FCT(A-C))
      C=.4520586281852374300*B
      Y=Y+.5346966299765921 5D-1*(FCT(A+C)+FCT(A-C))
```

```
      C=.38495133709715234D0*B
      Y=Y+.80039164271673110D-1*(FCT(A+C)+FCT(A-C))
      C=.29365897714330872D0*B
      Y=Y+.10158371326153296D0*(FCT(A+C)+FCT(A-C))
      C=.18391574949909010D0*B
      Y=Y+.11674626826917740D0*(FCT(A+C)+FCT(A-C))
      C=.62616704255734458D-1*B
      Y=B*(Y+.12457352290670139D0*(FCT(A+C)+FCT(A-C)))
      RETURN
      END
C
C
C
C     ******************************************************************
C             PROGRAM  TWO
C     ******************************************************************
C
C     NUMERICAL EVALUATION OF MODIFIED WKB SOLUTION FOR MASS FLUX -
C     PRESSURE RELATIONSHIPS
C
C     CALCULATION OF COEFFICIENTS U,U',V,V'
      IMPLICIT REAL*8(A)
      REAL*8 Y,F,X,PRMT,W
      DIMENSION Y(4),F(4),PRMT(5),AUX(16,4)
      COMMON W,A1,A2,AMN(4)
      EXTERNAL FCT,OUTP
C     AMN IS THE VECTOR OF MACH NUMBERS OF INTEREST
      AMN(3)=.835803 2D-1
      AMN(4)=.1165960D0
      A1=DSQRT(1.2D0**3)
      PRMT(1)=1.D-4
      PRMT(2)=AMN(4)**2
      PRMT(3)=.4D-4
      PRMT(4)=5.D-7
    2 FORMAT('0','W=',D10.4)
    4 FORMAT(1X,4D15.7)
      DO 10 J=1,19
      K=J
C     W IS THE FREQUENCY NO.
      W=K*1.D-2
      L=J-9
      IF(J.GT.10) W=L*1.D-1
C     DERIVATIVE VECTOR, INITIALLY CARRIES ERROR WEIGHTS
      F(1)=0
      F(2)=0
      F(3)=1.D0
      F(4)=0
C     INITIAL CONDITIONS
```

```
      Y(1)=1.D0
      Y(2)=0.D0
      Y(4)=-3.28612502824454 6D-2/W
      Y(3)=-.26295427621699 89D-5/W
      WRITE(6,2)W
      WRITE(6,4)Y(2),Y(1),Y(4),Y(3)
C     NUMERICAL INTEGRATION USING HAMMING'S METHOD
      CALL DHPCG(PRMT,Y,F,4,IHLF,FCT,OUTP,AUX)
   10 CONTINUE
      STOP
      END


      SUBROUTINE OUTP(X,Y,F,IHLF,NDIM,PRMT)
      IMPLICIT REAL*8(A)
      REAL*8 X,Y,F,PRMT,W
      COMMON W,A1,A2,AMN(4)
      DIMENSION Y(4),F(4),PRMT(5)
C     X IS THE MACH NO. (AM) SQUARED
   11 FORMAT(1X,5D15.7,I6)
   20 FORMAT('0',1X,3E20.4,D20.4,//)
      IF(IHLF.GE.8) GO TO 10
      IF(DABS(X-AMN(1)**2).LE.PRMT(3)) GO TO 4
      IF(DABS(X-AMN(2)**2).LE.PRMT(3)) GO TO 5
      IF(DABS(X-AMN(3)**2).LE.PRMT(3)) GO TO 6
      IF(DABS(X-AMN(4)**2).LE.PRMT(3)) GO TO 7
      GO TO 12
    4 AM=AMN(1)
      GO TO 9
    5 AM=AMN(2)
      GO TO 9
    6 AM=AMN(3)
      GO TO 9
    7 AM=AMN(4)
      GO TO 9
    9 CONTINUE
      WRITE(6,11)X,Y(2),Y(1),Y(4),Y(3),IHLF
      A5=-4.D0*A1/A2*X**1.25D0/(Y(1)**2+Y(3)**2)
      AN1=-A1/A2*(X**.25D0-.4D0*X/A2)+A5*(Y(1)*Y(2)+Y(3)*Y(4))
      AN2=-W*(1.D0+1.D0/AM)+A5*(Y(1)*Y(4)-Y(3)*Y(2))
      AN4=-AN2-W/AM
      AN3=-AN1
      AMPFAC=DSQRT((AN1**2+AN2**2)/(AN3**2+AN4**2))
      ANGLE=DATAN((AN2*AN3-AN1*AN4)/(AN1*AN3+AN2*AN4))
      PHASE=ANGLE*180./3.1415927
C     S IS THE MODIFIED STROUHAL NUMBER
      S=W*A2**2/DSQRT(AM**3)/A1
C     MAGNITUDE OF PHASOR PI/MU
```

```
      PIMU=1.4*AM**2*AMPFAC/(1.-AM**2)
      WRITE(6,20)S,PIMU,PHASE,AMPFAC
      RETURN
   10 CONTINUE
      WRITE(6,11)X,Y(2),Y(1),Y(4),Y(3),IHLF
   12 RETURN
      END


      SUBROUTINE FCT(X,Y,F)
      IMPLICIT REAL*8(A)
      REAL*8 X,Y,F,W
      DIMENSION Y(4),F(4)
      COMMON W,A1,A2,AMN(4)
      A2=1.D0+.2D0*X
      A3=2.D0-.08D0*X
      AAA=A3/16.D0/X/A2**2
      AAB=W/(A1*2.D0*X**1.25D0)
      AAC=.2D0/A2-1.2D0/X
      F(1)=Y(2)
      F(2)=Y(1)*AAA+Y(4)*AAB+Y(2)*AAC
      F(3)=Y(4)
      F(4)=Y(3)*AAA-Y(2)*AAB+Y(4)*AAC
      RETURN
      END
C
C
C
C     ****************************************************************
C           PROGRAM  THREE
C     ****************************************************************
C
C     MASS FLUX CALIBRATION OF THE HOT WIRE ANEMOMETER
C
      DIMENSION XX(126),YY(126),XO(39),YO(39)
      REAL*8 ITITLE(12)/'CARLSON
     1      HOT WIRE CALIBRATION                          '/,LABEL1/8H
     2  /
      REAL*8 X(39),FX(39),WI(39)/39*1./,Y(39),DELY(39),B(21),SB(21),TITL
     1E(10)/10*' '/
      REAL LABEL2/4H    /
      C=SQRT(53.34*531./1.4/32.2)
      PA=30.14
      PR=PA*70.73
      G=.2/.7
 1000 FORMAT(2F10.4)
 1500 FORMAT(' ',2F12.5)
 2000 FORMAT('0',3F12.4)
```

```
      M=0
      VO2=2.091**2
      DO 100  I=1,25
      READ(5,1000)DELP,V
      PO=PA+DELP/13.6
      P=(PO/PA)**G
      RHOV=PR/C*SQRT(P*(P-1.)/.2)
      WRITE(6,2000)DELP,V,RHOV
      M=M+1
      X(M)=V**2-VO2
      FX(M)=RHOV
      XO(M)=X(M)
      YO(M)=FX(M)
  100 CONTINUE
      DO 200  I=1,14
      READ(5,1000)DELP,V
      PO=PA+DELP
      P=(PO/PA)**G
      RHOV=PR/C*SQRT(P*(P-1.)/.2)
      WRITE(6,2000)DELP,V,RHOV
      M=M+1
      X(M)=V**2-VO2
      FX(M)=RHOV
      YO(M)=FX(M)
      XO(M)=X(M)
  200 CONTINUE
      KK=-8
      L=9
      DO 4000  N=1,2
      CALL LSQPL2(39,KK,X,FX,WI,Y,DELY,B,SB,TITLE)
      WRITE(7,3000)B
 3000 FORMAT(3D24.16)
      DO 5000  J=1,126
      XX(J)=.2*(J-1)
      YY(J)=POLY(B,L,XX(J))
      WRITE(6,1500)XX(J),YY(J)
 5000 CONTINUE
      CALL DRAW(30,XO,YO,1,1,LABEL1,ITITLE,5.,10.,0,0,2,2,6,10,1,LAST)
      CALL DRAW(126,XX,YY,3,0,LABEL2,ITITLE,5.,10.,0,0,2,2,6,10,1,LAST)
      KK=-9
      L=10
 4000 CONTINUE
      STOP
      END
```

```
      FUNCTION POLY(B,N,X)
C     EVAL OF POLYNOMIAL B(1)+ + +B(N)*X**(N-1) BY NESTING
      REAL*8 B(N),BNEST,POLY
      BNEST=B(N)
      K=N-1
      DO 1000 I=1,K
      BNEST=B(N-I)+BNEST*X
 1000 CONTINUE
      POLY=BNEST
      RETURN
      END
C
C
C
C     ************************************************************
C               PROGRAM  FOUR
C     ************************************************************
C
C     REDUCTION OF EXPERIMENTAL DATA
C
C     DYNAMIC DATA HAS BEEN NARROW BAND PASSED WITH GAIN SET AT 20 DB
C     SCALING IS CORRECTED FOR BY THIS PROGRAM
      IMPLICIT REAL*4 (M)
      REAL*8 B(9)
C     READ COEFFICIENTS OF HOT WIRE POLYNOMIAL
      READ(5,200)B
  200 FORMAT(3D24.16)
C     N IS THE NUMBER OF DATA POINTS
      N=20
C     MICROPHONE SENSITIVITY SENS
      SENS=10.06577
      G=.2/.7
C     STAGNATION TEMPERATURE TO
      TO=530.
C     ATM PRESSURE IN IN. MERCURY
      PA=30.02
C     HOT WIRE DC VOLTAGES
      VO=2.127
      V=4.016
      DELPO=4.
C     PO IS THE STAGNATION PRESSURE
C     FOR DELPO IN IN. WATER
      PO=PA+DELPO/13.6
C     PO=PA+DELPO   FOR IN. MERCURY
      DELPE=.1
C     PE IS THE EXIT PRESSURE
      PE=PA+DELPE/13.6
C     MACH NO. M
```

```
      M=SQRT(((PO/PE)**G-1.)/.2)
      CONST=1.4*M**2/(1.-M**2)
C     GAIN IS A CORRECTION OF 14 DB FOR BP FILTER
      GAIN=10.**.7
C     CORRECTION FOR RMS VOLTAGE
      RMS=SQRT(2.)/2.*GAIN
      VO2=VO**2
      X=V**2-VO2
C     MASS FLUX RHOV   (LBM/FT**2/SEC)
      RHOV=POLY(B,9,X)
      WRITE(6,500)M,RHOV
  500 FORMAT(' ','M='E12.7,'RHOV=',E12.7)
C     RHO IS THE EXIT DENSITY
      RHO=PO*70.73/53.34/TO*(PE/PO)**(1./1.4)
C     U IS THE EXIT VELOCITY
      U=RHOV/RHO
      DO 5000 I=1,N
C     READ IN FREQUENCY AND RMS VOLTAGES FOR PRESSURE AND MASS FLUX
      READ(5,1100)W,PPP,RRR
 1100 FORMAT(3F10.4)
      WRITE(6,1300)W,PPP,RRR
 1300 FORMAT('0',3E20.7)
      MAXV=(V+RRR/RMS)**2-VO2
      MAXRV=POLY(B,9,MAXV)
C     MU IS THE DIMENSIONLESS MASS FLUX PERTURBATION
      MU=(MAXRV-RHOV)/RHOV
C     PI IS THE DIMENSIONLESS PRESSURE PERTURBATION
      PI=PPP/SENS/PE
C     MODIFIED STROUHAL NO. S
      S=3.1415927*W*2.5/6./U
      PIMU=PI/MU
C     MBAR IS THE AMPLITUDE FACTOR
      MBAR=PIMU/CONST
      WRITE(6,2000)S,PIMU,PI,MU,MBAR
 2000 FORMAT('0',5E20.7)
 5000 CONTINUE
      STOP
      END

      FUNCTION POLY(B,N,X)
C     EVAL OF POLYNOMIAL B(1)+ + +B(N)*X**(N-1) BY NESTING
      REAL*8 B(N),BNEST,POLY
      BNEST=B(N)
      K=N-1
      DO 1000 I=1,K
      BNEST=B(N-I)+BNEST*X
 1000 CONTINUE
```

```
POLY=BNEST
RETURN
END
```

REFERENCES

1. Elrod, H. C., Jr., "The Theory of Pulsating Flow in Conical Nozzles," Journal of Applied Mechanics, v. 30, Trans. ASME, v. 85, Series E, p. 1-6, 1963.

2. Chiang, T., and others, "Analysis of Pulsating Flows in Infinite and Finite Conical Nozzles," Journal of Applied Mechanics, v. 36, Trans. ASME, Series E, No. 2, p. 159-170, June, 1969.

3. Oppenheim, A. K., and Chilton, E. H., "Publishing Flow Measurement - A Literature Survey," Trans. ASME, v. 77, p. 231, 1955.

4. Robertson, J. M., Hydrodynamics in Theory and Application, Prentice-Hall, 1965.

5. Shapiro, A. H., The Dynamics and Thermodynamics of Compressible Fluid Flow, v. 1, Ronald, 1953.

6. Morse, P. M., and Feshback, H., Methods of Theoretical Physics, v. II, McGraw-Hill, New York, p. 1093, 1953.

7. Ralston, A., and Wilf, H. S., Mathematical Methods for Digital Computers, Wiley, New York, 1960.

8. IBM, System/360 Scientific Subroutine Package (360A-CM-03X) Version III Programmer's Manual, Form H20-0205-3, IBM Corp., Poughkeepsie, New York, 1968.

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Documentation Center Cameron Station Alexandria, Virginia 22314	2
2.	Library, Code 0202 Naval Postgraduate School Monterey, California 93940	2
3.	Asst. Professor T. Houlihan, Code 59Hm Department of Mechanical Engineering Naval Postgraduate School Monterey, California 93940	1
4.	LT Edward J. Carlson 9220 Smith Avenue Baltimore, Maryland 21234	1
5.	Department of Mechanical Engineering Code 59 Naval Postgraduate School Monterey, California 93940	1

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Naval Postgraduate School Monterey, California 93940	2a. REPORT SECURITY CLASSIFICATION Unclassified
	2b. GROUP

3. REPORT TITLE
An Experimental Investigation of Pulsating Subsonic Flow in a Conical Nozzle

4. DESCRIPTIVE NOTES (Type of report and, inclusive dates)
Master's Thesis, September 1971

5. AUTHOR(S) (First name, middle initial, last name)
Edward J. Carlson

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
September 1971	53	8

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO.	
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d.	

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Naval Postgraduate School Monterey, California 93940

13. ABSTRACT

Amplitude and phase relationships between sinusoidal pressure disturbances in the exit plane of a conical nozzle and mass flux response were experimentally determined for the subsonic flow of air in the nozzle. The experimental results were compared with values predicted by using the modified asymptotic WKB method.

Experimental measurements were made with a constant temperature hot wire anemometer and a condenser microphone. Disturbance frequencies from 20 to 250 Hz were investigated for Mach numbers up to .34.

The experimental values of amplitude factor agree quite well with analytical predictions for all Mach numbers tested. Experimental values for phase angle differed from those predicted but exhibited a similar trend.

KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Conical Nozzle						
Pulsating Flow						
Amplitude and Phase Relationships						
Subsonic Nozzle Flow						

Thesis 132542
C233 Carlson
An experimental investigation of pulsating subsonic flow in a conical nozzle.